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Modal Logic and Ontological Arguments

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by

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In this thesis I will evaluate various modal ontological arguments for the existence of God. I will evaluate Kurt Gödel's modal ontological argument (as preserved by his student Dana Scott), and Anthony Anderson's emendation of it. I will conclude that both Gödel's argument and Anderson's emendation fail. I then propose a revised version of C'Zar Bernstein's ontological argument. I conclude that while this argument is not rationally compelling, it is more plausibly sound than not. Finally, I will demonstrate that some extant general objections to modal ontological arguments are unconvincing.

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Alex Yousif

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I. INTRODUCTION

Foremost among our modal headaches is [the] ontological argument.

—David Lewis[1]

By "ontological arguments" I mean a relatively unique class of arguments best characterized as *a priori* arguments for the existence of God; that is, arguments whose premises do not appeal to experiential observations. These arguments owe their name to Immanuel Kant and originate with St. Anselm of Canterbury (c. 1033-1109). Various philosophers, including: Thomas Aquinas, Rene Descartes, Gottfried Leibniz, and Immanuel Kant, have all commented on the merit of this class of arguments. Most find ontological arguments to be unsuccessful; yet, as Bertrand Russell pointed out, it is not very easy to pinpoint where exactly an ontological argument goes wrong.[2] Advances in modal logic in the last century have contributed to a new breed of ontological arguments—modal ontological arguments.

In this paper I examine and evaluate a few of these modal ontological arguments.[3] In particular, I focus on Kurt Gödel's modal ontological argument, C. Anthony Anderson's emendation of it, and my revised version of C'Zar Bernstein's argument. In chapter II, I evaluate Gödel's ontological argument and Anthony Anderson's emendation thereof. I conclude that both the original argument and the emendation are unsound. In chapter III, I advance my refined version of C'Zar Bernstein's ontological argument, concluding, first, that it is sound; second, that though probably sound it is not a

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[1] David Lewis, "Anselm and Actuality," *Noûs* 4, no. 2 (1970): 175.
[2] Graham Oppy, "Ontological Arguments," *Stanford Encyclopedia of Philosophy,* E.N Zalta, ed., accessed April 1, 2015, http://plato.stanford.edu/entries/ontological-arguments.
[3] Clearly, an analysis of *all* modal ontological arguments would be beyond the scope of this paper.

compelling argument for God's existence.[4] In Chapter IV, I consider four general objections to modal ontological arguments, but argue these objections do not succeed.

Since my aim is to examine the merits of just a few modal ontological arguments that have been proposed in the secondary literature, I generally set historical remarks aside. Accordingly, I am not interested in the historical overview of the discussion concerning ontological arguments (e.g., from Anselm to Descartes).[5] I believe contemporary formulations of the ontological argument tend to be better than their historical counterparts. One reason is the emphasis is no longer on what is conceivable (as it is in Anselm's argument(s)), but on what is metaphysically possible, and metaphysical possibility has nothing to do with our putative or actual conceptions. This effect, I believe, renders the contemporary modal formulations simpler. Contemporary modal formulations of the argument also benefit from logical rigor, and are less ambiguous than their counterparts in the historical world. For these reasons, I believe that if any types of ontological arguments have a chance at succeeding, they are modal ones. Having said this, I start by analyzing some of these modal ontological arguments.

[4] That is, it is not a "compelling argument" in the sense that reasonable people of good will can rationally disagree about the soundness of the argument. The argument is not so strong as to rationally compel belief in its soundness.

[5] For those interested, see: Alvin Plantinga, *The Ontological Argument: From St. Anselm to Contemporary Philosophers* (New York: Double Day Anchor, 1965); Graham Oppy, *Ontological Arguments and Belief in God* (New York: Cambridge University Press, 1995).

II. GÖDEL'S ONTOLOGICAL ARGUMENT

Kurt Gödel's Ontological Argument

Kurt Gödel, once considered the world's foremost mathematician and logician, is perhaps most famous for his incompleteness theorems.[6] What is less known about Gödel is that he offered his own ontological argument for the existence of God. In this chapter, I examine his modal ontological argument as preserved by his student Dana Scott, and the emendations of it given by C. Anthony Anderson. I conclude that both the original argument and the emendation given by Anderson are unsound.

While never published, there is evidence Gödel constructed his modal ontological argument *circa* 1941.[7] In 1970, Gödel showed it to his student Dana Scott. Scott in turn showed the argument to his student, and it eventually became a part of the wider scholarly community. I lay out Gödel's argument, as preserved by his student Dana Scott below.[8] Following precedent, the symbolic rendition I use is, with few exceptions, identical to Scott's notes (transcribed by Sobel). I lay out the (abbreviated) statements as found in Scott's notes, as well as the more rigorous formulations provided by Sobel in

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[6] His first incompleteness theorem states that "any consistent formal system *F* within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of *F* which can neither be proved nor disproved in *F*." His second incompleteness theorem states that "for any consistent system *F* within which a certain amount of elementary arithmetic can be carried out, the consistency of *F* cannot be proved in *F* itself." For more information on Gödel's Incompleteness Theorems, see "Gödel's Incompleteness Theorems," *Stanford Encyclopedia of Philosophy,* E.N Zalta, ed., accessed April 1, 2015, http://plato.stanford.edu/entries/goedel-incompleteness.

[7] Robert M. Adams, "Notes to 1970*" in *Kurt Gödel: Collected Works: Volume III: Unpublished Essays and Lectures*, ed. Solomon Feferman (Oxford: Oxford University Press, 1986), 388.

[8] John H. Sobel, *The Logic of Theism* (Cambridge: Cambridge University Press, 2009), 145-46. This is the same version that is evaluated by John H. Sobel in his "Logic and Theism."

numbered statements.[9] Following formal conventions, Definitions are listed first, followed then by Axioms before the important Theorems. Since theorems have to be proved, informal proofs are given and derived from only the relevant definitions, axioms, and inference rules. I assume the reader is familiar with standard first-order logical notation, but I add the following:

Abbreviation 0: The following serve as abbreviations for the terminology used below:

‘ϕ’ and ‘ψ’ are second-order variables ranging first-order predicates or properties,

‘Gx’ for a monadic first-order predicate ‘x is God-like’,

‘PX’ for a monadic second-order predicate ‘X is positive’, and

‘$\neg p$’ for ‘it is not the case that p is true’[10]

‘ϕ Ess. x’ for ‘ϕ is the essence of x’

‘NEx’ for ‘x necessarily exists’.

Definition 1: x is God-like if and only if x has all positive properties.

Def. 1: $Gx \leftrightarrow \forall\phi[\mathrm{P}\phi \rightarrow \phi x]$

(D1) $\Box\forall \mathrm{x}\,[Gx \leftrightarrow \forall\phi[\mathrm{P}\phi \rightarrow \phi x]]$

[9] Sobel explicitly binds the variables and adds a necessary operator in front of the formulas since if the formulas are true at all, they are necessarily true. For convenience, when writing the informal proofs, I will excise the necessary operator and use the original Gödelian axioms as preserved by Scott.

[10] On some views, $\neg\phi = \hat{x}[\neg\phi x] = \{x: \neg\phi x\}$

Definition 2: A property ϕ is an *essence* of x if and only if x has ϕ and ϕ entails all of x's properties.

Def. 2: ϕ Ess. $x \leftrightarrow [\phi x \ \& \ \forall\psi[\psi x \rightarrow \Box\forall y[\phi y \rightarrow \psi y]]$

(D2) $\Box\forall\phi \ \forall x \ [\phi$ Ess. $x \leftrightarrow [\phi x \ \& \ \forall\psi[\psi x \rightarrow \Box\forall y[\phi y \rightarrow \psi y]]]$

Definition 3: x necessarily exists if and only if every essence of x is necessarily exemplified.

Def. 3: $NEx \leftrightarrow \forall\phi \ [\phi$ Ess. $x \rightarrow \Box \ \exists x \ \phi x]$

(D3) $\Box\forall x \ [NEx \leftrightarrow \forall\phi \ [\phi$ Ess. $x \rightarrow \Box \ \exists x \ \phi x]]$

Axiom 1: For any property ϕ and its negation $\neg\phi$, either ϕ is positive or $\neg\phi$ is positive, but not both. That is, for any pair of properties ϕ and $\neg\phi$, *exactly one* is positive.

Ax.1: $[P\neg\phi \leftrightarrow \neg P\phi]$

(A1) $\Box\forall\phi \ [P\neg\phi \leftrightarrow \neg P\phi]$

Axiom 2: Any property entailed by a positive property is positive.[11]

Ax.2: $[P\phi \ \& \ \Box\forall x[\phi x \rightarrow \psi x]] \rightarrow P\psi$

(A2) $\Box\forall\phi\forall\psi \ [P\phi \ \& \ \Box\forall x[\phi x \rightarrow \psi x]] \rightarrow P\psi$

Axiom 3: The property of being God-like is positive.[12]

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[11] A property ϕ *entails* another property ψ if and only if $\Box\forall x \ [\phi x \rightarrow \psi x]$. Throughout this paper, I will speak interchangeably between a property's ϕ entailing a property ψ, and something's having ϕ entailing its having ψ.

[12] Technically speaking, Gödel's premise here is not well formed. "G" is a first-order predicate constant and not a proposition, so it needs a term—either a constant or variable—attached or "in

Ax.3: $\mathrm{P}G$

Axiom 4: If a property is positive, then necessarily, it is positive.

Ax.4: $\forall\phi\,[\mathrm{P}\,\phi \rightarrow \Box\,\mathrm{P}\,\phi]$

(A4) $\Box\forall\phi\,[\mathrm{P}\,\phi \rightarrow \Box\,\mathrm{P}\,\phi]$

Axiom 5: Necessary existence is a positive property.

Ax.5: PNE

Theorem 1: If a property is positive, then it is possibly exemplified.

Th.1: $\mathrm{P}\,\phi \rightarrow \Diamond\exists x\,\phi\,x$

(T1) $\Box\,\forall\phi\forall \mathrm{x}\,[\mathrm{P}\phi \rightarrow \Diamond\exists \mathrm{x}\,\phi\,\mathrm{x}]$

Theorem 2: If something is God-like, then the property of being God-like is an essence of that thing.

Th.2. $\Box\forall x\,[Gx \rightarrow G\,\mathrm{Ess}\,x\,]$

Theorem 3: Necessarily, the property of being God-like is exemplified.

Th.3: $\Box\exists xGx$

For ease of reference, the symbolic argument is succinctly rendered as follows:

Def.1: $\Box\forall \mathrm{x}\,[Gx \leftrightarrow \forall\,\phi\,[\mathrm{P}\,\phi \rightarrow \phi x]]$

Def.2: $\Box\forall\phi\forall x\,[\phi\text{ Ess. }x \leftrightarrow [\phi x\ \&\ \forall\psi[\psi x\rightarrow\Box\forall y[\phi y\rightarrow \psi y]]]$

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it". The reader is urged to keep this mind, although we will stick to the formalization given by Dana Scott.

Def.3: $\Box\forall x\, [NEx \leftrightarrow \forall\phi\, [\phi\ \text{Ess.}\ x \rightarrow \Box\, \exists x\, \phi x]]$

Ax.1: $\Box\forall\phi[P\neg\phi \leftrightarrow \neg P\phi]$

Ax.2: $\Box\forall\phi\forall\psi\, [P\phi\ \&\ \Box\forall x[\phi x \rightarrow \psi x]] \rightarrow P\psi$

Ax.3: PG

Ax.4: $\Box\forall\phi\, [P\phi \rightarrow \Box\, P\phi]$

Ax.5: P NE

Th.1: $\Box\forall\phi\, [P\phi \rightarrow \Diamond\exists x\, \phi x]$

Th.2: $\Box\forall x\, [Gx \rightarrow G\ \text{Ess}\ x\,]$

Th.3: $\Box\exists x Gx$

The argument is rendered in (higher-order) quantified modal logic. It makes use of one undefined primitive word—viz., "positive." Some have claimed that the argument goes through in **S5** (a widely accepted modal system)**.** [13] While this is true, **S5** is not necessary for the argument to go through. This is because Gödel's argument is logically valid in **Brouwer**, a weaker modal system abbreviated as '**B**'.[14] In what follows, I will briefly and informally explicate how the three theorems follow from the axioms and definitions.

Proof of Th.1: Assume for a *reductio* $P\phi$ and $\neg\Diamond\exists x\phi x$. By modal and quantifier negation, $\Box\forall x\neg\phi x$. But since everything is entailed by impossible properties, it follows that $\Box\forall x\, [\phi x \rightarrow \neg\phi x]$. So

[13] Stamatios Gerogiorgakis, "Does the Kind of Necessity which Is Represented by S5 Capture a Theologically Defensible Notion of a Necessary Being?" in *Ontological proofs today*, ed. Mirosław Szatkowski (Berlin: Ontos Verlag, 2012), 309.
[14] Since system S5 is a stronger modal system, every theorem of **B** is a theorem of **S5**. Most importantly, the characteristic modal axiom of **B**—viz., $\phi \rightarrow \Box\Diamond\phi$—is a theorem of **S5**.

it follows that $P\phi$ and $\Box\forall x\,[\phi x \rightarrow \neg\phi x]$. But given this and Ax.2, it follows that $P\,\neg\phi$. But given the left-handed conditional of Ax.1, it follows that $\neg P\,\phi$. So $P\,\phi$ and $\neg\,P\,\phi$. This is a contradiction. So reject the assumption: it is not the case that $P\,\phi$ and $\neg\Diamond\exists x\phi x$. Therefore, $\Box\forall\phi\,[P\phi \rightarrow \Diamond\exists x\,\phi\,x]$.

■

Proof of Th.2: Suppose, for conditional proof, that Gx. But recall that, by definition of essence, $\forall\phi\forall x[\phi \text{ Ess. } x \leftrightarrow \phi x\ \&\ \forall\psi[\psi x\rightarrow\Box\forall y[\phi y\rightarrow \psi y]]]$. So, $[G \text{ Ess. } x \leftrightarrow Gx\ \&\ \forall\psi[\psi x\rightarrow\Box\forall y[Gy\rightarrow \psi y]]$. Since we already have Gx as an assumption, the aim is to derive $\forall\psi[\psi x\rightarrow\Box\forall y[Gy\rightarrow \psi y]]$. Now, if we show that x has only positive properties, then we can show that all properties of x are entailed by G (since, G by definition entails all positive properties). Let ψ be an arbitrary property of x. Suppose that it is not the case that ψ is positive; then, by Ax.1, $\neg\psi$ is positive. But since by having Gx, x by definition has all positive properties, it follows that x would exemplify $\neg\psi$. Then, ψx and $\neg\psi x$. But it is manifestly impossible for anything to exemplify both some property and its negation. So reject the assumption: it is not the case that it is not the case that ψ is positive. Therefore, every property of a God-like being is positive. For by definition of a God-like being, necessarily, every property of a God-like being is positive. Further, by Ax.4, necessarily, every positive property is necessarily positive. So every property that x has is entailed by its instantiating the property of being God-like.

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Proof of Th.3: The theorem states that $\Box\exists xGx$. So if we can show that $\Diamond\exists xGx \rightarrow \Box\exists xGx$, and $\Diamond\exists xGx$, then we can derive $\Box\exists xGx$. But Th.1 and Ax.3 entail that $\Diamond\exists xGx$. So what remains is to show that $\Diamond\exists xGx \rightarrow \Box\exists xGx$. Now, if $\Diamond\exists xGx$, then by Ax.5, $\Diamond\exists x\,(Gx\ \&\ NEx)$—i.e., possibly, something is

God-like and every essence of that thing is necessarily instantiated. But by Th.2, G is an essence of whatever that has it— so $\Diamond\Box\exists x\, Gx$. But by the modal B axiom, it follows $\Box\exists x Gx$. So it has been shown that $\Diamond\exists x Gx \rightarrow \Box\exists x Gx$. But since $\Diamond\exists x Gx$, it follows that $\Box\exists x Gx$.

■

Clearly, the argument is valid, so the soundness depends on whether the axioms are true. In what follows, I examine the truth of the axioms.

An Examination of Gödel's Axioms

But before we evaluate the axioms, I address Gödel's use of the term 'positive'. As noted above, Gödel takes the word 'positive' to be primitive and leaves it (formally) undefined. However, he does briefly albeit ambiguously hint at the notions he is trying to capture with the word. Further, Gödel arguably borrowed at least the term from Leibniz, who discussed the ontological argument in terms of "simple and positive" properties.[15] But ambiguity will not do here since what he means by 'positive' is crucial to assessing whether or not some properties (e.g., necessary existence as he defines it), are indeed 'positive'. So what does he mean by 'positive'? The answer is not clear. Although from his terse notes, he seems to mean positive in either the morally aesthetic and axiological sense, or in the logical sense implying no privation (or both). If 'positive' is to be taken in the axiological sense, then it is not clear at all that the property of *necessarily existing* (as is traditionally defined in contemporary philosophy or by Gödel) is positive. What does having no contingent properties essentially have to do with what is morally appealing? However, if 'positive' is to be taken in the logical sense, then it produces an even more mysterious primitive word—viz., 'privation'. What exactly does it mean to

[15] Robert Adams, "Notes to 1970*", 389.

say that some property P does not imply some privation Q? It is far from clear. Given these terminological difficulties, it seems prudent to heuristically take "positive" to mean either "perfection" or "great-making property", following Robert Maydole and Alexander Pruss.[16]

My evaluation of Gödel's axioms centers on Axiom 1 (i.e., $\Box\forall\phi\,[P\,\neg\phi \leftrightarrow P\phi]$), which I believe is false because it implies that there is no pair of properties of the form ϕ and $\neg\phi$, such that neither ϕ nor $\neg\phi$ is positive. But it seems like there is some pair of properties ϕ and $\neg\phi$ such that they are both neutral properties, and hence not positive.[17] Such a pair of properties includes the property of being such that there are stones, and the property of not being such that there are stones. As Anderson points out, it seems like neither of these properties is positive. These are plausibly neutral properties, i.e., properties that are neither positive nor negative. [18]

Axiom 2 (i.e., $\Box\forall\phi\forall\psi[[P\,\phi\ \&\ \Box\forall x[\phi x \rightarrow \psi x]] \rightarrow P\,\psi]]$ is false. This is because there are some positive properties that entail neutral properties. Hájek gives a particularly poignant example of this.[19] Let a devilish object be an object having all negative properties. So the paradigmatic positive property of *being omnipotent*, e.g., entails the gerrymandered property of *being omnipotent or a devilish object*. But it seems

[16] Pruss defines a great-making property as a property that in no respect detracts from the greatness of any being that is it, but whose complement does; Alexander R. Pruss, "A Gödelian Ontological Argument Improved," *Religious Studies* 45, no. 3 (2009): 347. Maydole defines a perfection as a property that it is better to have than to lack; Robert Maydole, "The Ontological Argument," in *The Blackwell Companion to Natural Theology*, ed. William Lane Craig and J.P. Moreland (Malden: Wiley-Blackwell, 2009), 580.
[17] A neutral property is a property that is neither positive nor negative. So, obviously no neutral property is positive. Discerning what property is neutral will admittedly turn on intuition.
[18] C. Anthony Anderson, "Some Emendations of Gödel's Ontological Proof," *Faith and Philosophy* 7, no. 3 (1990): 291–303.
[19] Peter Hájek, "A New Small Emendation of Gödel's Ontological Proof," *Studia Logica* 71, no. 2 (2002): 150.

implausible that this disjunctive property is positive—it seems to be neutral. This constitutes a counterintuitive result of Ax.2. In fact, *every* positive property could work in a counterexample against Ax.2; this is because every positive property entails gerrymandered disjunctive properties that are plausibly neutral. Another reason for thinking that this axiom is false is because it would entail that every negative property entails an infinite number of positive properties. For any negative property, e.g., the property of *being devilish*, would entail positive disjunctive properties—e.g., the property of *being devilish or omniscient*. But this does not seem to be intuitively correct, and so Ax.2 is false. Maydole claims that although it is counterintuitive to affirm that disjunctive properties like the one above are positive, this does not amount to a refutation of the axiom.[20] His implicit reasoning here seems to be that if an axiom in the ontological argument entails a counterintuitive result, then that is not in and of itself a sufficient reason to reject the axiom. But surely this is incorrect; after all, axioms of modal ontological arguments rely heavily on intuition. For what basis do we have for affirming that certain properties, like the property of being God-like, omniscient, or omnipotent, are positive, or perfections (if we are to go with Maydole's lingo) if not intuition? Indeed, what basis do we have to say that the property of being a devilish being is negative other than intuition? It would be quite strange to claim that the proposition that **the property of being devilish is a positive property** is not false even though it entails a counterintuitive result.[21] The counterintuitive result here is sufficient to show that the proposition is false.

[20] Robert Maydole, "The Ontological Argument," in *The Blackwell Companion to Natural Theology*, ed. William Lane Craig and J.P. Moreland (Chichester, U.K.; Malden, MA: Wiley-Blackwell, 2009), 577.

[21] Throughout this paper, I use boldface font to mention the proposition expressed by a token declarative sentence. I follow Scott Soames's *Philosophical Analysis in the Twentieth Century*, 2vols (Princeton: Princeton University Press, 2003).

Likewise, and contrary to Maydole, the counter intuitiveness of Ax.2 *is* sufficient to show that it is more probably false than true.

Now, one might object to the above counterexample on the grounds that disjunctive properties cannot be instantiated. But this is controversial, not a very promising route, and not one that Gödel would have agreed with.[22] It seems that disjunctive properties can be generated from negative and conjunctive properties,[23] properties that are *prima facie* less problematic. For example, the disjunctive property of *being red or human* can be generated from the negative property of *not being not red and not human* (in a way analogous to how (R v H) can be derived from $\neg(\neg R \ \& \ \neg H)$ via DeMorgan's rule in sentential logic). So if one wants to deny that disjunctive properties can be instantiated, then one must also deny that negative and conjunctive properties can be instantiated, which does not seem to be plausible.

Axiom 3, the proposition that **the property of being God-like is positive**, appears true. Recall that for Gödel, to say that an entity is God-like is to say that it has all positive properties. And it seems that for any positive properties ϕ and ψ, their conjunction, viz., (ϕ & ψ) is positive. But it combinatorially follows from this that the property of having all positive properties is positive.[24]

Axiom 4 seems to be true. If some property is positive in the actual world, then it seems like it would be positive in all possible worlds. In order for the argument to go

[22] This is because Gödel believed in both negative and conjunctive properties.
[23] By 'negative' here I do not mean 'negative' in the sense used by in discussion of Gödel's argument, but in the general sense that corresponds to the negation operator in sentential logic. So, heuristically speaking, if the property of *being red* is P, the property of *being not red* would be ¬P.
[24] The case is spelled out in Bernstein's quote on page 20 of this paper.

through, it needs to be the case that there are no positive properties that are only positive properties contingently; every positive property must be a positive property necessarily. The proposition strikes me as intuitively obvious, and I see no special pleading here. The proposition that some property is positive has the same feature as many metaphysical propositions—viz., the feature of being true only if necessary true.[25] So if some property is positive, then it is necessarily the case that that property is positive.

Axiom 5 appears to be true—prima facie, the property of existing necessarily is a positive property. The axiom seems to amount to no more than the claim that the property of existing necessarily is a positive or perfective property seems to be a staple assumption of ontological arguments that goes at least as far back as Descartes.[26] But the axiom does 'not wear its content on its sleeve,' so to speak. By 'necessary existence' Gödel did not just mean what is usually meant by the phrase after the advent of Saul Kripke and Alvin Plantinga's work in modality—viz., existence across all possible worlds. Rather, recall that for Gödel x necessarily exists if and only if every essence of x is necessarily exemplified. Now, just like with necessary existence, Gödel's notion of essence is not the standard notion of essence today. Following Plantinga, many contemporary philosophers conceive of an essence as something that refers to the set of properties that that thing exemplifies in all possible worlds at which it exists.[27] But recall that for Gödel, a property ϕ is an essence x if and only if ϕ entails every property of x. The upshot of all of this is that for Gödel, something's having necessary existence implies that it has contingent-free

[25] Indeed, this is why Sobel adds necessary operators to the non-atomic axioms in Gödel's argument; they are formulas that if true, are necessarily true.
[26] René Descartes, *The Philosophical Writings of Descartes: Volume 2* (New York: Cambridge University Press, 1984), 83.
[27] Alvin Plantinga, *The Nature of Necessity* (New York: Oxford University Press, 1974), 70-88.

existence. We can say that something has contingent-free existence if and only if it does not have any properties contingently, but only necessarily. Given this understanding of necessary existence, Ax.5 loses its plausibility. Gödel's property of existing necessarily is not a positive property for the same reasons discussed in my refutation of Ax.3. It is plausible that everything that exists instantiates neutral properties. But if this is the case, then the instantiation of the property of being necessarily existent is impossible. And since it is an impossible property, it is not a positive property. For it seems like **Theorem 1** is true (even for extra-Gödelian reasons)—viz., a property is positive only if it is possibly instantiated. For there would be no sense in saying impossible properties, like the property of being numerically non-self-identical, are positive properties, despite the fact that they are instantiated in no possible world.

From the above evaluation, we have seen that three out of five of Gödel's axioms (viz., axioms 1, 2, and 5) are false. The argument is therefore unsuccessful. But in addition to the above critiques, Sobel argues that the conjunction of Gödel's axioms must be false, since the conjunction of the axioms entail that everything that exists, exists necessarily, and that every truth is a necessary truth—a result which is surely unacceptable.[28] As Sobel says, "given the generous interpretation of 'property' that is in force for the system, a God-like being would have properties that entailed the existence of every existent and the truth of every truth."[29] Sobel's argument can be summarized as follows:[30]

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[28] This result has been called 'modal collapse'; cf. Sobel, *Logic and Theism*. Modal collapse is unacceptable because obviously propositions like **I exist** are necessary truths!
[29] Sobel, *Logic and Theism*, 132.
[30] Sobel provides a formal proof for his modal collapse argument in *Logic and Theism*, 55.

Suppose that *j* exists, has the property G of being God-like, and has some property ϕ. Then by Th.2, it follows that G is the essence of *j*. But by definition of essence, it follows that G entails ϕ. Given this, and given that by Th.3, G is necessarily instantiated, it follows that G necessarily instantiates ϕ. But ϕ can be a property like the property of *being self-identical and such* ***that some (arbitrary) proposition Q is true***. So then, for every true proposition Q, Q is necessarily true. So, it follows from the conjunction of Gödel's axioms that every actually true proposition is necessarily true. But surely this is unacceptable! Therefore, the conjunction of Gödel's axioms is false—so at least one of the axioms must be false.

Taking Sobel's reasoning here to be inescapable, C. Anthony Anderson emends some of Gödel's axioms in an attempt to circumvent Gödel's modal collapse.[31] Recall that Ax.1 states the following:

Ax.1: $\Box \forall \phi\ [P \neg\phi \leftrightarrow \neg P\ \phi]$

Now, Ax. 1 is a biconditional premise, and so Anderson breaks it down into the following two material conditionals, starting with the "only if" part:

Ax.1.a: $\Box \forall \phi\ [P \neg\phi \rightarrow \neg P\ \phi]$

The logically equivalent contrapositive of this, which may be more intuitively appealing, says that if a property is positive then its negation is not positive—i.e., $\Box \forall \phi$ $[P\ \phi \rightarrow \neg P \neg\phi]$. But this is logically equivalent to $\Box \forall \phi [\neg P\ \phi \ \mathrm{v}\ \neg P\ \neg\phi]$. This entails that it cannot be the case that a property and its negation are *both* positive. In other words, it states that for any property ϕ and its negation $\neg\phi$, *at most* one is positive.

[31] C. Anthony Anderson, "Some Emendations of Gödel's Ontological Proof," *Faith and Philosophy* 7, no. 3 (1990): 291–303.

Consider, now, the "if" part of the biconditional:

Ax.1.b: $\Box \forall \phi\ [\neg P\ \phi \rightarrow P \neg \phi]$

This states that for any property ϕ and its negation $\neg\phi$, *at least* one is positive.

Now, Ax.1.a seems to be intuitively obvious. Clearly if a property is positive then its negation cannot be positive. Indeed, Anderson attempts to motivate Ax.1.a by way of more basic "intrinsic preferability" propositions, but it is not clear that those propositions are more intuitively obvious than Ax.1.a itself.[32] So Ax.1.a seems true. But what about Ax.1.b?

Ax.1.b is false for reasons elucidated above (in my evaluation of Ax.1). It rules out the possibility that there could be a pair of properties ϕ and $\neg\phi$ that are neutral properties, such as the properties of *being such that there is a stone and not such that there is a stone*.[33] Now, one might object here that Gödel means only to quantify over intrinsic properties in his ontological argument. And since pairs of properties like *being such that there is a stone and being not such that there is a stone*, are paradigmatically extrinsic properties (at least when had by non-stones), they do not constitute a counterexample to Ax 1.a. But this response will not do, for even if Gödel meant only to quantify over intrinsic properties, there are plausibly intrinsic pairs of properties that are plausibly neutral—e.g., the properties of being in space, or not being in space. There does not seem to be anything positive about being in space, or not being in space. Hence, we have good reason to believe that Ax. 1.b is false.

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[32] Ibid., 295.
[33] Ibid.

But recall that in arguing against Ax.1, we were really objecting against Ax.1.b., and did not object to Ax 1.a. This suggests that Ax.1 can still be used in a revised Gödelian argument. Indeed, that is just what Anderson does; he retains Ax.1.a in his emendation of Gödel's proof, and excludes Ax 1.b. So Ax.1 has become

Axiom 1*: If a property is positive, then its negation is not positive.

In addition to refining Ax.1, Anderson also refines Gödel's definitions to read as follows:

Definition 1*: *x* is God-like if and only if *x* has as essential properties those and only those properties that are positive.[34]

Definition 2*: ψ is an essence of *x* if and only if for every property ϕ, *x* has ϕ essentially if and only if ψ entails ϕ. In other words, ψ is an essence of *x* if and only if ψ entails every essential property of *x*.

Definition 3*: *x* necessarily exists if and only if every essence of *x* is necessarily exemplified [by *x*].

Anderson's Axiom 1* is the only emended axiom whose wording differs from the original axioms. The second and fourth of Anderson's axioms are unaffected by the above definitional refinements, and so still express the same propositions as Ax.2 and Ax.4. However, Anderson's third and fifth axioms make use of the new definitions (though are worded identically) and so no longer express the same propositions as Ax.3 and Ax.5. Therefore, to differentiate, I label Anderson's third and fifth axioms Ax.3*, and Ax.5*, respectively.

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[34] Anderson's use of 'essential properties' here is identical to the standard use in the literature today. On this conception, a property ϕ is essential to *x* if and only if *x* instantiates ϕ at every world in which it exists. See Ibid.

So is Anderson's emendation an improvement on Gödel's argument, and is the emended argument sound? Well, Anderson's emendations do avoid Sobel's modal collapse. Furthermore, he has proved the logical validity of his argument, so the soundness of the argument will turn on whether or not his axioms are true.

As we have already discussed, Ax.1* is logically weaker than Ax.1, and does seem to be an improvement. After all, Ax.1 is false for reasons discussed above, whereas Ax.1* seems true; it is hard to imagine how the negation of a positive property can be positive. Ax.2 is the same axiom used by Gödel, and so is false for the same reasons elucidated above. Ax.3* says that the property of being God-like* is a positive property. But notice that Anderson's definition of a God-like being is bolder than Gödel's. For on Gödel's definition, x is a God-like being if and only if x (i) has all positive properties. But on Anderson's definition, x is a God-like* being if and only if x (i) has all positive properties, (ii) has them essentially, and (iii) has no other properties essentially. As we have already seen, Ax.3 is true. It follows from the very plausible premise that the conjunction of any positive properties ϕ and ψ is itself a positive property. But Ax.3* seems to be false—for it seems plausible to suppose that any being, and so *a-fortiori* a God-like being, must have some neutral properties essentially. The property of being numerically self-identical is an essential property of everything that exists. But this property seems to be a (necessarily) neutral property.[35] From this it follows that the property of being a God-like* object is an impossible property (i.e., a necessarily uninstantiated property)—for there can be no being that has as essential properties *only* positive properties. But no impossible property is a positive property. This is because a

[35] A corollary of Ax.4 for neutral properties is plausible. That is, it seems that if a property is neutral, then necessarily, it is neutral.

positive property is a property that an object in virtue of having it, somehow increases its greatness or 'positiveness'. However, impossible properties can never increase some thing's greatness—for they are necessarily uninstantiated. So it follows that Ax.3* fails. If one has qualms with my assertion that the property of *being numerically self-identical* is a neutral property, and takes it to be positive, one can substitute other plausibly neutral and essentially had gerrymandered properties in its stead—e.g., the property of being an abstract or concrete object. Given this, it seems that Ax.3 is probably false.[36] Ax.4 is the same as Gödel's axiom, and so is true for all the reasons stated above. Ax.5* states that the property of existing necessarily* is a positive property. Now, by definitions 2* and 3*, it follows that *x* necessarily exists if and only if every property that entails every essential property of *x* is necessarily exemplified by *x*. So is this property, viz. NE*, a positive property? Well, it is not clear that this is a positive property. I see no motivation to affirm that NE* is a positive property. Notice that it is consistent with the definitions to suppose that an object that instantiates NE* is not God-like*, and does not have any positive properties whatsoever. Likewise, it is consistent with the definitions that *x* has some negative or neutral properties. Given this result, it becomes clearer that NE* is not a positive property. For suppose that *x* has a very large amount of essentially had negative properties. Then, *x's* necessarily exemplifying all its essences entails that it necessarily exemplifies its very large set of negative properties $\{N_1 \ldots N_n\}$. But surely any property

[36]As an aside, it is interesting to note that if one accepts Th.1 (which is entailed by Ax.1* and Ax.2), and the proposition that **there are neutral properties essentially had by all objects**, then it logically follows that Ax.3 is false. This is because if the proposition that **there are neutral properties essentially had by all objects** is true, then the property of being God-like* is an impossible property. But this in conjunction with Th.1 entails that it is not the case that the property of being God-like* is a positive property. Hence ¬Ax.3.

that implies such a set of properties is not a positive property! Therefore, NE* is not a positive property—so Ax.5* is false.

In conclusion, we have seen that Gödel's ontological argument is unsound, and that Anderson's emendation also fails.

III.THE BERNSTEINIAN MODAL ONTOLOGICAL ARGUMENT

In the previous chapter I argued that Gödel's ontological argument, and Anderson's emendation of it, are unsound arguments. In this chapter, in light of arguments published by C'Zar Bernstein, I advance my revised Bernsteinian ontological argument.[37]

Consider the following modal ontological argument:[38]

(1) Possibly, God, i.e., a being that has all perfections, exists.

(2) Existing necessarily is a perfection.

(3) Therefore, God exists.

The argument is quite simple, and I believe I may, without undue impropriety, call this a Cartesian-Leibnizian Argument.[39]

However, as it stands, the argument as stated by Bernstein is technically invalid, as (2) is missing a necessarily operator in front of it.[40] Similar points (e.g., ill-formed formulas) detract from the overall force of Bernstein's presentation. So his argument requires modification. Appropriately, I have refined and modified his definitions, premises, and proofs; I have also dropped reference of *perfections* in lieu of *great-making properties*. Below I revise Bernstein's argument.

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[37] C'Zar Bernstein, "Giving the Ontological Argument Its Due," *Philosophia* 42, no. 3 (2014): 665–79; C'Zar Bernstein, "Is God's Existence Possible?," *Heythrop Journal* 56, no. 2 (2014):1-9.
[38] C'Zar Bernstein, "Giving the Ontological Argument Its Due," 665.
[39] This is because Descartes affirmed that existing necessarily is a perfection, and Leibniz pointed out that Descartes' argument is incomplete insofar as it needs the premise that **possibly, God exists**. Bernstein's argument thus builds on the contribution of both of these thinkers.
[40] This is may just be an editorial mistake, for it is natural to assume that if something is a great making property (GMP), then necessarily, it is a GMP (although he talks in terms of perfections).

Before stipulating the following definitions and providing symbolic abbreviations, I begin with an important semantico-philosophical remark. Let '*F*' be a first-order predicate constant '_ is *F*'. Following Richard Montague's use of λ-abstraction, I introduce the following notational convention.[41] I let is '$\lambda x.Fx$' be the property expressed by the '*F*' predicate.[42] Suppose '*a* is *F*' is true. Normally, we would say that '*Fa*' is true if and only if the referent of '*a*' belongs to the extension of '_ is *F*'. However, when we λ-abstract, we create a property of *being F*. Under this non-extensional analysis, '*a* is *F*' is true if and only if *a* has the property of *being F,* which is now written as '$\lambda x.Fx[a]$'. The difference between '*Fa*' and '$\lambda x.Fx[a]$' is that the first is purely extensional and involves no reference to properties; however, the second is intensional and the λ-abstraction produces a property.

Notice, however, this process may be applied to higher-order properties. Suppose we wanted to talk about a property of properties—e.g., *being a monadic property*—that we may symbolize using λ-abstraction as '$\lambda\phi.M\phi$'. This is a property of properties that $\lambda x.Fx$ has resulting in the symbolization '$\lambda\phi.M\phi[\lambda x.Fx]$'. I read the preceding expression as: "*being F* has the property of *being a monadic property*."[43]

While this method is technically correct, it is overly pedantic. Performing λ-abstraction on every predicate in order to talk about the property expressed is

[41] For Montague's contribution, see David R. Dowty et al., *Introduction to Montague Semantics* (Dordrecht, Holland, 1981).
[42] As Anderson says in "Some Emendation's to Gödel's Proof,": "The technically minded will thus wish to note that it is in effect assumed that anything is counted as a property which can be defined by 'abstraction on a formula'" (p. 292).
[43] If I had written '$\lambda\phi.O\phi[F]$' I would have written something false, since '*F*' is a predicate and the predicate does not have the property of *being a monadic property*.

cumbersome; so, following convention,[44] I use the notation '^' and attach it to the front of the predicate. The resulting expression is the property expressed by the predicate. Accordingly, the notation '^*F*' stands for $\lambda x.Fx$ or *being F*. Likewise, '^*M*' stands for a second-order property *being a monadic property*, and the property ^*F* has it. So, '^*M*(^*F*)' is true. Thus, for any *n*-order predicate we may create an *n*-order property using the '^' notation. With this in mind, I introduce the following abbreviations and definitions:

Abbreviation 0: The following serve as abbreviations for the terminology used below:

> '$\mathcal{G}$^(^*O*)' is a second-order property standing for '$\mathcal{G}$(^O) is a great making property',[45]
>
> '*P*' for the first-order predicate '_ is perfect',
>
> '^$\mathcal{G}$' is the second-order predicate '_ is great making',
>
> '□' is the standard modal necessity operator restricted to models with an accessibility relation that is reflexive and symmetric.[46]
>
> '^*N*' is $\lambda x. \Box \exists y[y = x]$; that is, the property of *necessarily existing*.

Definition 1: A property ^*O* is a *great making property* (GMP) if and only if for any two distinct objects, *x* and *y*, that have (virtually) the same properties $\hat{}F_1, \hat{}F_2, \ldots, \hat{}F_n$, and *x* has ^*O* and *y* does not, then *x* is greater than *y*.[47]

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[44] I follow L. T. F. Gamut, *Logic, Language, and Meaning, Volume 2: Intensional Logic and Logical Grammar* (Chicago: Chicago University Press, 1991), §4.

[45] To be clear, a second-order property is a property of properties. It is higher-order intensional logic. See my earlier references.

[46] A modal model with an accessibility relation that is reflexive and symmetric is the modal system known as **B**, which I addressed above.

[47] I take the 'greater than' relation to be primitive. This will no doubt be problematic for some, but I think we have some intuitive grasp regarding what objects are greater than others. For example, most people, theists or not, would agree that if God were to exist, then he would be

Definition 2: For any object, x, x is *perfect* if and only if for any property, $^\wedge\phi$, if $^\wedge\phi$ is a $^\wedge\mathcal{G}$, then x has $^\wedge\phi$—i.e., $\Box\forall x[Px \leftrightarrow \forall^\wedge\phi[^\wedge\mathcal{G}(^\wedge\phi) \rightarrow {}^\wedge\phi x]]$

Definition 3: For any object, x, x is *imperfect* if and only if there is a property, $^\wedge\phi$, that is $^\wedge\mathcal{G}$ and it is not the case that x has $^\wedge\phi$—i.e., $\Box\forall x[Ix \leftrightarrow \exists^\wedge\phi[^\wedge\mathcal{G}(^\wedge\phi) \,\&\, \neg^\wedge\phi x]]$

Definition 4: A property $^\wedge O$ is a *lesser making property* (LMP) if and only if for any two distinct objects, x and y, that have (virtually) the same properties $^\wedge F_1$, $^\wedge F_2$,…,$^\wedge F_n$, and x has $^\wedge O$ and y does not, then y is greater than x.

My revised Bernsteinian modal ontological argument can be symbolized as follows, where (1)-(3) are my premises:

(1) $\Diamond\exists xPx$

(2) $\Box^\wedge\mathcal{G}(^\wedge N)$

(3) $\Box[\exists x(Px \,\&\, {}^\wedge Nx) \rightarrow \Box\exists xPx]$

From **Definition 2**, it follows that:[48]

(4) $\Box[[\exists xPx \,\&\, {}^\wedge\mathcal{G}(^\wedge N)] \rightarrow \exists x(Px \,\&\, {}^\wedge Nx)]$

From (1), I maintain

(5) $\exists xPx$

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greater than humans. This demonstrates that most people, at least, have some intuitive understanding of the relation.

[48] Assume (4) is false. Let the formula be true in $w_@$, where I get $\exists xPx$ and $\mathcal{G}^\wedge(^\wedge N)$ but not $\exists x(Px \,\&\, {}^\wedge Nx)$—i.e., $\neg\exists x(Px \,\&\, {}^\wedge Nx)$. Suppose a is the value of 'x' in '$\exists xPx$' giving us 'Pa' and '$\mathcal{G}^\wedge(^\wedge N)$'. From Definition 1 (in $w_@$), let 'Px' be assigned the same value to its variable 'x', which was just obtained. Thus, I now have the consequent of Definition 2, $\forall^\wedge\phi[^\wedge\mathcal{G}(^\wedge\phi) \rightarrow {}^\wedge\phi a]$. Let us assign the value of '$^\wedge N$' to 'ϕ' providing $\mathcal{G}^\wedge(^\wedge N) \rightarrow {}^\wedge Na$. We have the antecedent giving us $^\wedge Na$. Conjoining 'Pa' and '$^\wedge Na$' in first-order logic it follows that $\exists x(Px \,\&\, {}^\wedge Nx)$. This is a contradiction; therefore, (4) is true.

is true in some world w_1. From (2), (3), (4) it follows that (6), (7), and (8) are true in the same world w_1,

(6) $\hat{}\mathcal{G}(\hat{}N)$

(7) $\exists x(Px \,\&\, \hat{}N\, x) \rightarrow \Box\exists xPx$

(8) $[\exists xPx \,\&\, \hat{}\mathcal{G}(\hat{}N)] \rightarrow \exists x(Px \,\&\, \hat{}Nx)$

From the conjunction of (5) and (6), I obtain the following in w_1:

(9) $\exists xPx \,\&\, \hat{}\mathcal{G}(\hat{}N)$

I may then derive, in w_1, the consequent of (7) from hypothetical syllogism via (7), (8) and (9):

(10) $\Box\exists xPx$

But, from (10) and the accessibility relation from the modal system **B**, we know

(11) $\exists \mathrm{xPx}$

Corollary 1: The property of *being perfect* is the property of *having all GMPs*.[49]

Corollary 2: The property of *being imperfect* is the property of *not having all GMPs*.[50]

Since the argument is valid, its soundness turns on whether the premises are true. Premises (5)-(11) all follow by the logical rules of inference. Premise (4) is true in virtue of the definition of a perfect object. Recall that a perfect object is an object that has all GMPs. So it must be the case that if something is perfect and the property of *necessarily existing* is a GMP, then there exists a necessarily existing perfect object (the proof for this

[49] *Proof*: Follows from Definition 2.
[50] *Proof:* Follows from Definition 3.

was also provided in fn. 44). Premise (3) seems to be manifestly true; for clearly it must be the case that if there is a perfect being that exists necessarily, then necessarily, this perfect being exists (for the property of existing necessarily is just the property of existing in all possible worlds).

So that leaves us with two premises—(1) and (2). The argument rests crucially on these two premises. Now, (2) follows from the following sub-argument, which relies on premises (12) and (13):

(12) $\Box\forall{}^\wedge\phi[{}^\wedge\mathcal{G}({}^\wedge\phi)\rightarrow \Box{}^\wedge\mathcal{G}({}^\wedge\phi)]$

(13) ${}^\wedge\mathcal{G}({}^\wedge N)$

Let '$^\wedge N$' be the value of the variable in (12), giving us:[51]

(14) ${}^\wedge\mathcal{G}({}^\wedge N) \rightarrow\Box{}^\wedge\mathcal{G}({}^\wedge N)$

Plainly then, from this and (14), which I take to be more probable than not, I obtain:

(15) $\Box{}^\wedge\mathcal{G}({}^\wedge N)$

Notice that premise (12) is a corollary of Gödel's **Axiom 4**, a Gödelian axiom that we concluded is more probably true than false. It seems that if any property is a great-making property in the actual world, then it is a great making property in all worlds. Further, (13) appears to be more probably true than false. For the property of *necessarily existing* is a plausible candidate for a GMP. It seems like for any two distinct objects, x and y, that have (virtually) the same properties ${}^\wedge F_1$, ${}^\wedge F_2$,…,${}^\wedge F_n$, and x has ${}^\wedge N$ and y does not, then x is greater than y. [52] So the argument for (2) is sound and (2) is true.

[51] Given that $\mathcal{R}(w_@, w_@)$, by **B**.
[52] This admittedly rests on intuition, but this does not seem to be problematic—for at bottom all arguments rest on certain fundamental truths that we just intuit to be true. We come to find out

So what remains in order to show that a perfect being exists is to show that (1) is true. Arguably, Bernstein's most significant contribution to the literature is the two novel arguments for the possibility premise.[53] I believe that the fundamental motivations for both of his arguments for $\Diamond\exists xPx$ are practically the same,[54] so I will just argue for a revised version of one of these arguments, his so-called *compossibility argument*.[55] The compossibility argument that he offers has three axioms. But the third of his axioms—viz., '$\forall\phi\ \forall\psi\ [\Box\neg\exists x[\phi x\ \&\ \psi x] \rightarrow \Box\forall x[\phi\ x \rightarrow \neg\psi x]]$', is a theorem of second-order logic (i.e., it is a logical truth in second-order logic), so it really does not do much work in the argument.[56]

Additionally, I prove that the second premise follows from my **Definition 3** and (12), but the proof is annexed to an appendix for readability. This is an original result since Bernstein seems to gloss over it. The argument can be rendered as follows:

(16) $\forall{}^\wedge\phi[\Box\forall x[{}^\wedge\phi x \rightarrow Ix] \rightarrow \neg{}^\wedge\mathcal{G}({}^\wedge\phi)]$

(17) $\forall{}^\wedge\phi\forall\psi\ [[\Box\forall \mathrm{x}[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x]\ \&\ {}^\wedge\mathcal{G}({}^\wedge\psi)\ ] \rightarrow \Box\forall \mathrm{x}[{}^\wedge\phi x \rightarrow Ix]]$[57]

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that the property of *necessarily existing* is a GMP in the same way that we find out that omnipotence and omnibenevolence are GMPs—viz., by intuition. Although some arguments have been advanced for why existing necessarily is a GMP (e.g., in C'Zar Bernstein's "Giving the Ontological Argument its Due", 8-10), it is not clear at all the premises of the arguments used are more probable than the proposition that existing necessarily is a GMP.

[53] C'Zar Bernstein, "Is God's Existence Possible?," 2; C'Zar Bernstein, "Giving the Ontological Argument its due," 670.

[54] Both are fundamentally motivated by the following propositions (translated into my preferred definitions): **The property of being imperfect is a LMP; the conjunction of any GMPs ϕ and Ψ is itself a GMP**; **GMPs do not entail LMPs**. But the compossibility argument, unlike Bernstein's other argument, does not need the premise that some property is neutral.

[55] C'Zar Bernstein, "Is God's Existence Possible?," *The Heythrop Journal* 56, no. 2 (2014): 1.

[56] Ibid., 2.

[57] The scope of the necessary operator in (17) is rather ambiguous in Bernstein, where it appears as premise (2) of the compossibility argument (cf. Ibid.,2). This is because Bernstein's (2), symbolized in my notation as '$\forall{}^\wedge\phi\forall{}^\wedge\psi\ [[\Box\forall \mathrm{x}[[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x]\ \&\ {}^\wedge\mathcal{G}({}^\wedge\psi)\ ] \rightarrow \Box\forall \mathrm{x}[{}^\wedge\phi x \rightarrow Ix]]$', has

From (16) and (17), I may prove

(18) $\forall{}^\wedge\phi\forall{}^\wedge\psi[\ [{}^\wedge\mathcal{G}({}^\wedge\phi)\ \&\ {}^\wedge\mathcal{G}({}^\wedge\psi)\ ] \rightarrow \Diamond\exists x[{}^\wedge\phi x\ \&\ {}^\wedge\psi x]]$

Proof: See Appendix B for the derivation.

■

The argument is valid, so the soundness of the argument turns on the truth of (16) and (17). Premise (17) follows from Def. 3 (the definition of an imperfect being) and the plausibly true Gödelian premise (12). The proof for this is found in Appendix A. The reasoning behind the proof is quite clear: if *x*'s instantiating ϕ entails that *x* lacks some great making property, and whatever is a great making property is necessarily a great making property, then it *must* be the case that if *x* instantiates ϕ, *x* is imperfect. So the crucial premise is (16). Premise (16) states that if having a property ϕ is sufficient for the imperfection of anything it instantiates, then ϕ is *not* a GMP. The argument for (16) is as follows: assume, for conditional proof, that having ϕ is sufficient for the imperfection of anything in which it inheres. Since the property of being imperfect is very plausibly a LMP, i.e., a property that detracts from and does not add to the greatness of any being in which it inheres, it follows that something instantiates ϕ only if it instantiates a LMP. But it seems that a property that has the higher-order property of entailing a LMP cannot be a GMP. For that higher-order property is plausibly only a property of LMPs, and by definition no LMP is a GMP. So ϕ is not a GMP. Therefore, if some property ϕ is

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an extra bracket before the '$^\wedge\phi x$'. So the necessary operator could be interpreted to range over just $\forall x[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x]$, or $\forall x[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x]$ *and* ${}^\wedge\mathcal{G}({}^\wedge\psi)$. In private correspondence with Bernstein, he has related to me that he intended to have the necessary operator range over only $\forall x[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x]$, which would signify property entailment. This interpretation is also supported by intra-textual evidence, insofar as his deduction assumes that the necessary operator ranges over just $\forall x[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x]$.

sufficient for the imperfection of any being in which it inheres, then it is not a GMP—i.e., (16) is true.[58] Since the only substantive premise, viz. (16), seems to be more plausibly true than false, the revised compossibility argument is plausibly sound. Therefore, the conclusion, (18), is true.

Now (18) represents the fact that if any properties $^\wedge\phi$ and $^\wedge\psi$ are GMPs, then it is possible for there to exist something that instantiates ϕ and Ψ. However, this is not quite (1), or $\Diamond\exists x Px$, and so, this argument is not in and of itself an argument for (1), although it is most of the argument for it. Bernstein's conclusion only shows that GMPs are pairwise compossible.[59] He is aware of this objection and states that "it does not succeed", ostensibly because he believes that conceptual analysis of (18) will lead us to believe that it expresses the same proposition as premise (1) of the main argument above. He needs some additional substantial premise to get from (18) to (1), as can be seen by at least the following reasoning that he utilizes to demonstrate that his compossibility argument does not just show that GMPs are pairwise compossible.[60]

> Suppose there are exactly four great making properties and call them P_1, P_2, P_3, and P_4. So far, the [compossibility] argument above will show that P_1 and P_2 are compossible. It will also show that P_3 and P_4 are compossible. So, plausibly, **the conjunctive property of having** $\lambda x(P_1x \,\&\, P_2x)$ **is a great making property**, call it J*.[61] The conjunctive property $\lambda x(P_3x \,\&\, P_4x)$ is also a perfection, call it K*…Hence, $\lambda x(P_1x \,\&\, P_2x)$.and $\lambda x(P_3x \,\&\, P_4x)$., both of which are great making

[58] C'Zar Bernstein "Is God's existence possible?," 3.
[59] Bernstein speaks of perfections, but I will speak of GMPs to retain consistency.
[60] I say 'at least' because his definition of a perfect being is bolder than mine insofar as a perfect being for him must have all GMPs *essentially* (and lack all LMPs essentially). So he needs to prove more things than I do.
[61] I have inserted my terminology and λ-notation here for consistency.

properties, are compossible. But if they are compossible then it is coherent to suppose that they both be instantiated together in a being. So, plausibly, $\lambda x(P_1x$ & P_2x & P_3x & $P_4x)$ is a perfection…Suppose there are four more perfections, P_5, P_6, P_7, and P_8. The argument will show that P_5 and P_6 are compossible, so that $\lambda x(P_5x$ & $P_6x)$ is a perfection. It will also show that P_7 and P_8 are compossible, so that $\lambda x(P_7x$ & $P_8x)$ is a perfection. Thus, given the argument, $\lambda x(P_5x$ & $P_6x)$ is compossible with $\lambda x(P_7x$ & $P_8x)$, which entails that $\lambda x(P_5x$ & P_6x & P_7x & $P_8x)$ is a perfection. It will then show that $\lambda x(P_1x$ & P_2x & P_3x & $P_4x)$ is compossible with $\lambda x(P_5x$ & P_6x & P_7x & $P_8x)$. This can be done even if there are infinitely many perfections, $\{P_1 \ldots Pn, P_{n+1} \ldots\}$ because we can still have the conjunction of P_1 and (P_2 & P_3 & ... Pn . . .). The argument will show that the latter conjunction is compossible with P_1. Thus, the premises of [the compossibility argument] are true only if all perfections are compossible [emphasis is mine].[62]

So the hidden premise here seems to be that **if some properties ϕ and Ψ are GMPs, then their conjunction is a GMP**.[63] From this, we can, as Bernstein notes, combinatorially derive the conclusion that having all GMPs is a GMP—i.e., **GP**. So what of this additional premise? Well, it is certainly a substantive premise insofar as it does not follow analytically from (21). However, it does seem to be intuitively appealing, for it seems that the conjunction of any GMPs is itself a GMP. That is, it seems like the only reason one would even think of denying this premise is if one thinks that gerrymandered

[62] Ibid.,3-4. The crucial premise in this reasoning is analogous to the reasoning found in Kurt Gödel's original Ax.3, which is different from the Ax.3 that Dana Scott preserved in his notes. Gödel contended that the conjunction of any great making properties.

[63] Symbolically this can be represented as follows: $\forall{^\wedge}\phi\forall{^\wedge}\Psi\ [{^\wedge}G({^\wedge}\phi)\ \&\ {^\wedge}G({^\wedge}\Psi) \rightarrow \lambda x[{^\wedge}\phi x\ \&\ {^\wedge}\Psi x]$.

properties, and so conjunctive properties, are impossible properties. But this is highly controversial to say the least; indeed, I would wager that very few metaphysicians would endorse the claim that conjunctive properties are impossible properties. It seems like gerrymandered properties can be generated in the same way that more logically complex formulas can be generated from logically simpler formulas. However, it is beyond the scope of this paper to defend this thesis. Therefore, the proposition that **GP** is true. This, in conjunction with (19), logically entails that that premise (1) of the ontological argument is sound. The proof for this is as follows:

Let "^P" be the value of both $^\wedge\phi$ and $^\wedge\psi$. It follows from (18) that:

(19) $[(^\wedge\mathcal{G}\,(^\wedge P)\ \&\ ^\wedge\mathcal{G}\,(^\wedge P)) \rightarrow \Diamond\exists x\,(^\wedge Px\ \&\ ^\wedge Px)]$

Assume as a premise:

(20) $^\wedge\mathcal{G}(^\wedge P)$

From (20), it follows per logical equivalence that (21) is true:

(21) $^\wedge\mathcal{G}(^\wedge P)\ \&\ ^\wedge\mathcal{G}(^\wedge P)$

And then from (19) and (21), I obtain:

(22) $\Diamond\exists x(^\wedge Px\ \&\ ^\wedge Px)$

which is logically equivalent to:

(23) $\Diamond\exists x(^\wedge Px)$

So it seems that we have a more plausibly sound than unsound argument for premise (1) of this ontological argument. The ontological argument is therefore sound.

There exists a perfect being—i.e., a being that that has all GMPs. Though a perfect being is not quite God, it is only a short step from here to the conclusion that God exists. It is plausible that many of the properties classically attributed to God—e.g., omnipotence, omniscience, omnibenevolence, etc., are GMPs. And so there is such a being with these properties. So God exists.[64]

Though I have argued that the argument is plausibly sound, it is not compellingly sound. Fundamental propositions in this line of reasoning, e.g., the propositions that **necessary existence is a GMP**, and **GMPs do not entail LMPs,** are based on intuitions that rational people of good will can disagree on. However, hardly any philosophical arguments for substantive conclusions are rationally compelling, and it seems to be that this argument is a very good argument for God's existence.

[64] As an aside, it is noteworthy that even if one denies that **GP**, one can get interesting conclusions just by affirming (18) and that each pair of properties $\{\hat{}N, \phi\}$ is a GMP. From this it will follow that a being that has $\hat{}N$ and ϕ exists. So, given that omnipotence is a GMP, it will follow (by **B**) that a necessarily existing omnipotent being exists. Given that omniscience is a GMP, it will follow that a necessarily existing omniscient being exists. And so on and so forth.

IV. GENERAL OBJECTIONS TO MODAL ONTOLOGICAL ARGUMENTS

Metaphysical and Logical Possibility are Identical

Some claim that modal ontological arguments for the existence of God are futile because metaphysical possibility (sometimes called 'broadly logical possibility' *a lá* Plantinga) just *is* logical possibility. On this view, since metaphysical and logical possibility are the same, it is logically possible that God does not exist if and only if it is metaphysically possible that God does not exist. And it is claimed that since it is clearly logically possible that God does not exist, after all, the proposition that **it is not the case that God exists** does not entail a contradiction,[65] it follows that it is metaphysically possible that God not exist. But this contradicts the conclusion of modal ontological arguments, which is that it is metaphysically necessary that God exist. In other words, this is the argument: Logical possibility is metaphysical possibility. But then a proposition is logically possible if and only if it is metaphysically possible. It is clearly logically possible that God does not exist. Therefore, it is metaphysically possible that God does not exist. But this contradicts the result of modal ontological arguments that conclude that it is metaphysically necessary that God exist. Therefore, modal ontological arguments are unsound.

One way to diffuse this objection is to deny that God's existence is logically contingent. Although some philosophers would argue that God's existence is logically necessary,[66] this does not seem to be true. For it seems that the negation of the proposition that **God exists**, does not entail a contradiction, solely in virtue of the

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[65] Richard Swinburne, "What Kind of Necessary Being Could God Be?" in *Ontological Proofs Today*, ed. Miroslaw Szatkowski (Berlin: Ontos Verlag, 2012), 345.
[66] Brian Leftow, "Swinburne on Divine Necessity," *Religious Studies* 46, no. 2 (2010): 141–62.

meanings of the words ‘God exists’, and (classical) logical axioms. So does this mean that modal ontological arguments are hopeless? I do not think so. There is another, and in my opinion quite plausible, way to diffuse the argument—viz., by denying that logical and metaphysical possibility are identical. It does not seem to me that logical and metaphysical possibility are the same. The set of propositions that are metaphysically possible seem to me to be a proper subset of the set of propositions that are logically possible. In other words, there are propositions that are logically possible but metaphysically impossible. I take the following to be instances of propositions of this sort: **it is permissible to torture babies for fun**; **one ought to do some action *A*, but one cannot do *A*; something is contingent, possibly has an explanation, but has no explanation; something *x* begins to exist at a time t_x, there is no time prior to t_x, *x* possibly has a cause, but *x* does not have a cause.** Nothing about logic and the meanings of the words in the proposition that **one ought to do some action *A*, but one cannot do *A*** entails a contradiction. But it doesn’t seem like this proposition is metaphysically possible. Surely, if one ought do some action *A*, then one must be able to do *A*. And surely this cannot fail to be the case! So we have a proposition that is logically possible but metaphysically impossible. These are counterexamples to the view that logical and metaphysical possibility are one and the same thing, although a detailed defense that each proposition is indeed a counterexample is beyond the scope of this paper. So I do not believe that this general objection against modal ontological arguments is successful.

Peter Van Inwagen's General Objection

Peter Van Inwagen offers a general objection to modal ontological arguments.[67] According to Van Inwagen, a set of properties is an *ontic* set if (and only if) it satisfies the following two conditions:[68]

(a) It contains the property of *existing necessarily*

(b) Possibly, something exists which instantiates all of its members essentially.

Van Inwagen then defines an *ontic* argument to be an argument that has exactly one premise of the form ***s* is an ontic set**, and one conclusion of the form ***s* is instantiated**. Now, he takes (modal) *ontological* arguments to be a species of ontic arguments, and ones that are about ontic sets that have at least the property of being necessarily existent, and the property of being concrete, as members.[69] Since every (modal) ontological argument is an argument for the existence of a being that is necessarily existing and concrete, a minimal ontological argument is as follows:

1. The set of properties $\{N,C\}$is an ontic set.

2. Therefore, $\exists x(Nx \ \& \ Cx)$.

Now, ontological arguments about ontic sets with more robust members—e.g,. $\{N$, C, omnipotence, omnibenevolence…$\}$ will be sound arguments only if (1) is true. But (1) is true only if (b*) Possibly, something exists which instantiates the property of existing necessarily and the property of existing concretely, essentially, is true.

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[67] Peter Van Inwagen, "Ontological Arguments," *Noûs* 11, no. 4 (1977): 375–95.
[68] Ibid., 377.
[69] Van Inwagen, though unsure about how to exactly characterize the notion of a concrete entity, nevertheless takes it be a non-abstract entity.

Van Inwagen's self-described "bold" thesis is that we cannot know whether or not (b*) is true or false, apart from divine revelation. He believes this because he believes that "[To show that *N* and *C*] are compatible, we should have to construct a formally valid argument having 'Something has both *N* and *C* as its conclusion and show that the conjunction of the premises of the argument is possibly true."[70]

He goes on to remarkably assert that we can neither show that *N* and *C* are compossible or incompossible because "the task of finding the required ancillary premises and of demonstrating that they have the required modal status is (in both cases) impossible"![71]

But van Inwagen hardly gives an argument for why this must be the case; indeed, the reader is left with the impression that he offers the invalid argument that since he has never seen any such sound argument, there not only is no such argument, but there couldn't possibly be one. However, this is hardly a convincing reason to accept his quite bold claim. So van Inwagen's general objection to modal ontological arguments has not been shown to succeed in its task. Indeed, there is good reason to believe that van Inwagen's objection has not only not been shown to succeed, but has been demonstrated to fail.

For I think one can indeed demonstrate that an ontic set consisting of at least members *N* and *C* is instantiated. I believe that the refined Bernsteinien ontological argument that I offered is sound and so does show that possibly, a being that has all GMPs exists. But, as I explained earlier, it is only a short step from there to the

[70] Ibid., 383.
[71] Ibid., 383.

demonstration that N^* and C are compossible.[72] The soundness of this ontological argument entails that there exists a perfect being that has the property of existing in all possible worlds accessible to the actual world. But it seems extremely plausible that the property of being omnibenevolent is a GMP—viz. a property that is necessarily better to have than to lack. So N^* and the property of being omnibenevolent, O, are compossible. But the latter property entails the property of being concrete—for nothing but a concrete individual can be omnibenevolent. Therefore, it follows that N^* and C are compossible. Now, it is important to note that although the revised Bernsteinian ontological argument, in addition to the premise that **omnibenevolence is a GMP**, entails that N^* and C are compossible, this is not sufficient to show that Van Inwagen's objection fails. For we have not shown that it is possible that something have N^* and C essentially, and so have not shown that this string of reasoning would count as an ontological argument in van Inwagen's sense. But surely a string of reasoning such as this *should* qualify as an ontological argument, irrespective of whether or not it shows that this being has its GMPs essentially. So van Inwagen should extricate the qualifier "essentially" from (b). But in any case, van Inwagen's objection as it stands still fails. This is because it seems very plausible that the property of having GMPs essentially is itself a GMP. And so every perfect being will instantiate this property, and hence every perfect being will have all its GMPs essentially. Given this and the aforementioned, it follows that a perfect being that has the GMPs N^* and O essentially, exists. Therefore, a perfect being that has N^* and C

[72] I say 'N^*' because the conception of necessary existence that I make use of is less bold than Van Inwagen and Bernstein's conceptions. On my operative conception, for a being to necessarily exist is for it to exist at all possible worlds *accessible to the actual world*. So a being could be a necessary being in my sense, but fail to be a necessary being in the sense stipulated by either Van Inwagen or Bernstein. I suspect that Van Inwagen would not mind refining criterion (a) to say the following: "[Set *s*] contains N or N^*." So I will assume as much in this discussion.

essentially, exists. So what Peter Van Inwagen deemed as an impossible task is not only a possible task, but an actually done task.

The Kantian Objection: "Existence is not a Predicate."

Perhaps the most famous general refutation of ontological arguments is inspired by Immanuel Kant (1724-1804). Indeed, a thesis about ontological arguments can hardly ignore this Kantian objection; so we will not here. The Kantian objection to ontological arguments can be summed up by the (now famous dictum) that "existence is not a predicate."[73] However, on the face of it, this claim is false, since the word 'predicate' is a grammatical term of art. But to say that existence is not a grammatical predicate would seem to be wrongheaded. For example, in the sentence 'The Earth exists', the rules of the English language specify that 'The Earth' is the grammatical subject in the sentence, and that the verb "exists" is the grammatical predicate. Here the verb 'exists' is predicated of 'The Earth'. So if one interprets the Kantian dictum to imply that in English sentences of the form 'The *x* exists', the verb 'exists' does not function as a predicate of '*x*', then the dictum is just false. However, this is not what the Kantian means when she says that 'existence is not a predicate'. By 'a predicate', she does not mean a grammatical predicate, but 'a property'. Put differently, the Kantian dictum says that existence is not a property; that is, the referents of singular terms, properly speaking, do not have the property of existing. This Kantian thought survives into the contemporary era, and has been given a more rigorous and logical construal by Gotleb Frege. Frege contended that existence is not a first-order property of individuals, like, *being human*. Rather, existence

[73] This comes from Kant who says the following: "'Being' is obviously not a real predicate; that is, it is not a concept of something which could be added to the concept of a thing" (B626). Immanuel Kant, The Critique of Pure Reason, H. Caygill, ed. Translated by N. K. Smith (New York: Palgrave Macmillan, 2003).

is a second-order property, one that does not apply to concrete individuals, but sets. On this Fregan view, which is very much in the Kantian spirit, sentences of the form '*x* exists' are different from sentences of the form '*x* is human' or '*x* is brilliant'. In the former type of sentences, the property picked out by 'exists' is not a property of *x,* but a property of the set of *x*-like things. It is a property which says that the set of *x*-like things has at least one existent member—in other words, 'exists' is as an existential operator here. So to symbolize 'Barack Obama exists' is just to say that '$\exists x(x$=Barack Obama)'.[74]

But this Kantian objection does not seem to be relevant to (most) modal ontological arguments for God—and certainly not the ones that have been discussed in this paper. For technically speaking, neither Gödel's argument, nor my refined Bernsteinian argument, presuppose that existence is a first-order property of individuals or a second-order property of sets. These arguments do not speak about existence *simplicter*, but of *existing necessarily*—the argument only assumes that existing necessarily is a first-order property of something or another, irrespective of what. It is the property of existing necessarily that is asserted as a positive property or a great-making property. Given this, if the Kantian objection is to have any chance of working against arguments like this, it must be refined. So perhaps one can, in the Kantian spirit, argue that existing necessarily cannot be a property of individuals, but of sets of properties. Sets of properties, if instantiated, are instantiated either contingently or necessarily. Ostensibly, the way to argue for this would be to argue that if existence is not a property of individuals, then neither is existing necessarily. However, this is questionable, for it does seem that on an intuitive level, it does make sense to ascribe necessary existence to

[74] If one believes that 'Barack Obama' is really a definite description and not a proper name, then one can replace 'Barack Obama' with a the appropriate description.

some individual (zero-order) objects—e.g., numbers. To use Kantian language, the concept of necessarily existing is something that could be added to the concept of a thing, and so the property of *existing necessarily* seems to be a real property. Therefore, Kantian objections, if successful at all against any modal ontological argument, do not succeed against the ones discussed here.

The Objection from Error Theory about Valued Properties

Another general objection that one might advance against modal ontological arguments strays from an error theory about valued properties. An error theorist about valued properties would deny that there are any such things as valued properties—for her, there are no properties that are good, positive, great-making, perfections, bad, etc. There are only value-free neutral properties. If the error theorists are right here, then standard modal ontological arguments, and certainly the ones discussed in this thesis, fail—for they all assert that some properties (e.g., the property of *being God-like*, or the property of *being perfect*) are, in some sense, valued properties.

I do not believe this is a good (pun intended!) general objection to modal ontological arguments. Indeed, it seems that everyone who is not a moral nihilist, and that is almost everyone, should have a problem with this type of objection. For if one is a moral realist, then one believes that there are some actions that are good or bad, and some people that are good or bad.[75] But if this is the case, then clearly people are good or bad in virtue of instantiating some good or bad properties—i.e., in virtue of instantiating valued properties (otherwise there would be no reason why some are good and some are bad!). Therefore, on moral realism, there are valued properties. So moral realists, which

[75] Although arguing for moral realism is beyond the scope of this thesis.

are most people, should have a problem endorsing the error theorist's objection. Furthermore, moral realism is in fact true. Clearly there are some actions, e.g., torturing innocent babies for fun, that are wrong; and clearly there are some people, e.g., Adolf Hitler, who are bad people. Therefore, valued properties exist.

It also seems that the error theorist must not only believe that there are no valued properties in the actual world to object to modal ontological arguments, but she must also believe the stronger claim that there are no valued properties in any possible world! Error theory about valued properties seems to be true only if necessarily true. This is because it seems that if it is possible that property ϕ is a valued property, then it is necessarily the case that ϕ is a valued property. But it seems obvious that there *could* be valued properties. However, this plausibly entails that there are valued properties. Therefore, the general objection to modal ontological arguments from error theory about valued properties is as failure.

V. CONCLUSION

In this thesis, we have examined three modal ontological arguments for the existence of God: Kurt Gödel's argument (as preserved by Dana Scott), C. Anthony Anderson's emendation of this argument, and my revised version of C'Zar Bernstein's argument. We have seen that Gödel's modal ontological argument for the existence of God is unsound. We have also seen that although Anthony Anderson's emendation of Gödel's argument is indeed an improvement, it is still a failure. With respect to the Bernsteinian modal ontological argument, we concluded that although the argument is not compelling, it is nonetheless plausibly sound. We have also examined four general objections that are advanced against modal ontological arguments and have found them wanting.

APPENDICES

A. Proof of Imperfection Premise

Proof: From the definition of an imperfect being, viz., $\Box\forall x[Ix \leftrightarrow \exists{}^\wedge\phi[{}^\wedge\mathcal{G}({}^\wedge\phi) \,\&\, \neg{}^\wedge\phi x]]$,

and the Gödelian premise that $\Box\forall\phi[{}^\wedge\mathcal{G}({}^\wedge\phi) \rightarrow \Box{}^\wedge\mathcal{G}({}^\wedge\phi)]$, I prove the following:

(17) $\forall{}^\wedge\phi\forall{}^\wedge\psi\, [[\Box\forall x[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x] \,\&\, {}^\wedge\mathcal{G}(\psi)] \rightarrow \Box\forall x[{}^\wedge\phi x \rightarrow Ix]]$

Proof: Assume for a reductio that (17) is false, so (1) is true:

(1) $\exists{}^\wedge\phi\exists{}^\wedge\psi\, [[\Box\forall x[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x] \,\&\, {}^\wedge\mathcal{G}(\psi)] \,\&\, \neg\Box\forall x[{}^\wedge\phi x \rightarrow Ix]]$

Assign the values of 'F' to ϕ and 'B' to ψ, giving us three conjunctions which I have simplified:

(2) $\Box\forall x[{}^\wedge\mathrm{F}x \rightarrow \neg{}^\wedge\mathrm{B}x]$

(3) ${}^\wedge({}^\wedge\mathrm{B})$

(4) $\neg\Box\forall x[{}^\wedge\mathrm{F}x \rightarrow Ix]$

After modal and quantifier negation, implication, and DeMorgan, I assume $\mathcal{R}(w_{@}, w_1)$

giving us the truth in w_1 (assigning the values of '*a*' to x):

(5) ${}^\wedge\mathrm{F}a \,\&\, \neg Ia$

From (2), the same accessibility relation and assignment to x:

(6) ${}^\wedge\mathrm{F}a \rightarrow \neg{}^\wedge\mathrm{B}a$.

Simplifying (5) we obtain:

(7) ${}^\wedge\mathrm{F}a$

(8) $\neg Ia$

From (7) and (6), we obtain in w_1:

(9) $\neg{}^\wedge\mathrm{B}a$

Suppose we utilize a Gödelian result such that:

(10) $\Box\forall\phi[{}^\wedge\mathcal{G}({}^\wedge\phi) \rightarrow \Box{}^\wedge\mathcal{G}({}^\wedge\phi)]$

Assume $\mathcal{R}(w_@, w_@)$, when 'B' is assigned the value of the variable of (10), we obtain in $w_@$:

(11) ${}^\wedge({}^\wedge B) \rightarrow \Box{}^\wedge\mathcal{G}({}^\wedge B)$

From (3) and (11),

(12) $\Box{}^\wedge({}^\wedge B)$

From the definition of Imperfection,

(13) $\Box\forall x[Ix \leftrightarrow \exists{}^\wedge\phi[{}^\wedge\mathcal{G}({}^\wedge\phi) \,\&\, \neg{}^\wedge\phi x]]$

Assume $\mathcal{R}(w_@, w_1)$, giving us the truth in w_1 (assigning the values of "a" to x):

(14) $Ia \leftrightarrow \exists{}^\wedge\phi[{}^\wedge\mathcal{G}({}^\wedge\phi) \,\&\, \neg{}^\wedge\phi a]$

From (8) it follows that in w_1:

(15) $\neg\exists{}^\wedge\phi[{}^\wedge\mathcal{G}({}^\wedge\phi) \,\&\, \neg{}^\wedge\phi a]$

Using $\mathcal{R}(w_@, w_1)$, from (12) we may obtain in w_1:

(16) ${}^\wedge({}^\wedge B)$

Conjoining (9) and (16) in w_1:

(17*) $\wedge(\wedge B)$ & $\neg\wedge$Ba

By existential-introduction from (17*) on the second-order constant "B", the following holds in w_1:

(18) $\exists\wedge\phi[\wedge\mathcal{G}(\wedge\phi)$ & $\neg\wedge\phi a]$.

Since (15) and (18) are inconsistent, the resulting conjunction is a contradiction, which was what was wanted. It therefore follows that (1) is false, which is equivalent to maintaining the truth of (17).

■

B. Proof that any GMPs ϕ and ψ are Compossible

Proof: From (16) and (17), stated below, I prove the following result:

(18) $\forall{}^\wedge\phi\forall{}^\wedge\psi[[{}^\wedge\mathcal{G}({}^\wedge\phi) \;\&\; {}^\wedge\mathcal{G}({}^\wedge\psi)\,] \rightarrow \Diamond\exists x[{}^\wedge\phi x \;\&\; {}^\wedge\psi x]]$

(16) $\forall{}^\wedge\phi[\Box\forall x[{}^\wedge\phi x \rightarrow Ix] \rightarrow \neg{}^\wedge\mathcal{G}({}^\wedge\phi)]$

(17) $\forall{}^\wedge\phi\forall{}^\wedge\psi\,[[\Box\forall x[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x] \;\&\; {}^\wedge\mathcal{G}(\psi)] \rightarrow \Box\forall x[{}^\wedge\phi x \rightarrow Ix]]$

Proof: Assume, for *reductio* that (16) and (17) are true, but (18) is not. Notice the negation of (18) is second-order logically equivalent to the following:[76]

(1) $\exists{}^\wedge\phi\exists{}^\wedge\psi[[{}^\wedge\mathcal{G}({}^\wedge\phi) \;\&\; {}^\wedge G({}^\wedge\psi)] \;\&\; \Box\forall x\,[{}^\wedge\phi x \rightarrow \neg{}^\wedge\psi x]]$

Introducing 'F' as a value for ϕ and 'B' as a value for ψ in (1), we obtain:

(2) $[[{}^\wedge\mathcal{G}({}^\wedge F) \;\&\; {}^\wedge\mathcal{G}({}^\wedge B)] \;\&\; \Box\forall x\,[{}^\wedge Fx \rightarrow \neg{}^\wedge Bx]]$

Simplifying the conjunction, and the first conjunct once more, we obtain

(3) ${}^\wedge\mathcal{G}({}^\wedge F)$

(4) ${}^\wedge\mathcal{G}({}^\wedge B)$

(5) $\Box\forall x\,[{}^\wedge Fx \rightarrow \neg{}^\wedge Bx]$

Assign the value of ϕ in (16) 'F', giving us:

(6) $[\Box\forall x[{}^\wedge Fx \rightarrow Ix] \rightarrow \neg{}^\wedge\mathcal{G}({}^\wedge F)]$

Assign the value of 'F' to ϕ and 'B' to ψ in (17), giving us:

(7) $[[\Box\forall x[{}^\wedge Fx \rightarrow \neg{}^\wedge Bx] \;\&\; {}^\wedge\mathcal{G}({}^\wedge B)] \rightarrow \Box\forall x[{}^\wedge Fx \rightarrow Ix]]$

[76] Justification: QN twice, Implication once, DeMorgan's, DN, Modal Negation once, QN once, De Morgan's, Implication, Double Negation.

Conjoining (4) and (5) gives us:

(8) $\Box\forall x\ [\hat{}Fx \rightarrow \neg\hat{}Bx]\ \&\ \hat{}\mathcal{G}(\hat{}B)$

From (7) and (8), it follows via Modus Ponens:

(9) $\Box\forall x[\hat{}Fx \rightarrow Ix]$

From (6) and (9), and again via Modus Ponens, we obtain:

(10) $\neg\hat{}\mathcal{G}(\hat{}F)$

Since (3) and (10) are inconsistent, the resulting conjunction is a contradiction, which was what was wanted. It therefore follows that (1) is false, which is equivalent to maintaining the truth of (18).

■

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